Assess the Complexity of an RM Synthesis Spectrum:

Introduction

This report provides an algorithm to assess whether an RM Synthesis Spectrum, (P, ϕ), is complex or simple. This forms module 2.5 in the POSSUM pipeline, as outlined in POSSUM Reports #5 and #7. The input are provided by module 2.4 (RMCLEAN), and the output is an integer; 0 for a simple spectrum, and other values for complex behaviour.

For the purposes of this module, a *simple* RM spectrum is defined as one that shows only a single, Faraday thin source. Faraday thin sources are ones where the polarized radiation is being Faraday rotated by a foreground screen, and there is little or no mixing between the emitting and rotating plasmas. An RM spectrum that is not simple is considered *complex*. Figure 1 shows two examples of a complex RM synthesis spectrum. The top shows two components that are individually Faraday thin, but happen to be along the same line-of-sight (or along different lines of sight within the same beam). The bottom spectrum is a Faraday thick source, where there is mixing between the emitting and rotating plasmas. In these plots, and in all of the tests of the algorithm presented below, we assumed observations with ν=1150-1450 MHz, and 300, 1-MHz channels.

Theory

After a series of tests over a wide range of physical models and parameters, we have identified the second moment of the model components produced by RM-CLEAN as a sensitive metric of deviations from a simple spectrum. These model components can be seen as green crosses in Fig. 1, which show the absolute value of the complex components used to clean the RM spectrum. The version of RM-CLEAN used for these tests was the basic Högbom CLEAN (Högbom 1974) used by Heald et al. (2009). Following the notation in POSSUM Report #7, the real and imaginary parts of the model components for each RM (ϕ) channel are given by $\mathrm{Re}_M(\phi)$ and $\mathrm{Im}_M(\phi)$. The actual quantity we are interested in is the square-root of the second moment, $\Delta\phi_M$, about the average ϕ (given by the first moment, $<\phi>$, of the clean component distribution), so that $\Delta\phi_M \equiv \sqrt{<(\phi - <\phi>)^2>}$, which is calculated by:

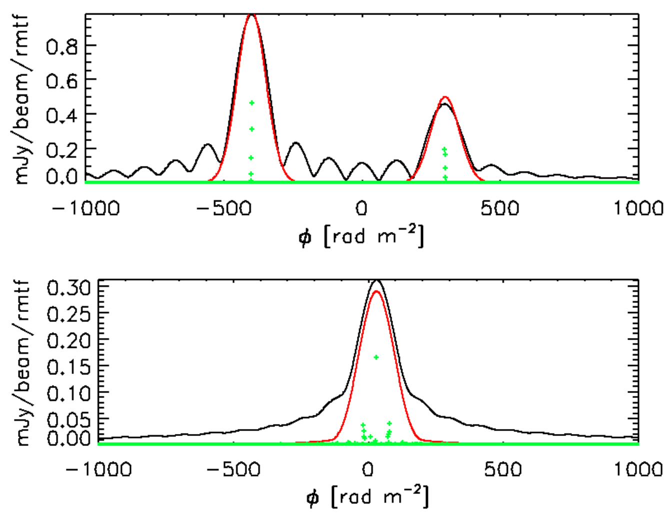

Figure 1: Top: RM spectrum of two Faraday thin components at -400 and +300 rad m^{-2}; Bottom: Faraday thick source with $\Delta\phi$=20 rad m^{-2}. Black lines are the dirty spectrum, red are the cleaned spectrum, and the green points are clean components.

$$\Delta\phi_M = K^{-1}\sum_{i=1}^{N_{RM}}(\phi_i - <\phi>)^2\sqrt{Re_M(\phi_i)^2 + Im_M(\phi_i)^2} \tag{1}$$

where the normalization, K, is given by

$$K = \sum_{j=0}^{N_{RM}}\sqrt{Re_M(\phi_i)^2 + Im_M(\phi_i)^2} \tag{2}$$

and $< \phi >$ is the first moment given by

$$<\phi> = K^{-1}\sum_{i=1}^{N_{RM}}\phi_i\sqrt{Re_M(\phi_i)^2 + Im_M(\phi_i)^2} \tag{3}$$

For the limiting case of a single clean component in channel i, the only non-zero value is at $\phi_i = <\phi>$ leading to $\Delta\phi_M = 0$. An example of the methods used to test the application of this metric can be seen in Fig. 2. We simulated a two component RM-spectrum, where the dominant component is placed at $\phi_1 = 0$ with an amplitude of 1 and a intrinsic polarization angle of $\chi_1 = 0$; the amplitude, rotation-measure (ϕ_2), and polarization angle (χ_2) of the second component was allowed to vary. Figure 2 shows a colour plot of $\Delta\phi_M$ as a function of ϕ_2 and χ_2 (the amplitude of the second component in this plot has been fixed at 0.5). This metric behaves exactly as we would like; it takes a single value ($\Delta\phi_M = 0$) everywhere in this parameter space corresponding to a simple spectrum ($\phi_2 = \phi_1 = 0$), and does not take that value anywhere else (i.e., we cannot get "fooled" into thinking a spectrum is simple when it is actually complex). This metric has been tested throughout the two component parameter space, as well as on Faraday thick emission where a single source is extended in the ϕ direction, and has shown the same behaviour.

Two simplifications of these tests, the lack of instrumental noise and the fact that all the sources were placed at pixel centres (in ϕ space), means that even simple sources will show some spread in the distribution of clean components when looking at real data. It is therefore necessary to introduce a threshold, $\Delta\phi_M = \epsilon_M$, below which we consider a measured spectrum to be simple. We are currently exploring the nature of this threshold, but it will likely be a function of other inputs to this module, such as the signal-to-noise of the dominant peak relative to S_0, the signal-to-noise threshold for detection (see POSSUM Report #7).

Algorithm

The algorithm is as follows: 1) Compute $\Delta\phi_M$ from equation 1; 2) Compute ϵ_M from the input data (algorithm to be determined at a later date); 3) If the computed $\Delta\phi_M$ is less than ϵ_M, return a 0; 4) If $\Delta\phi_M \geq \epsilon_M$, return another integer whose value will depend on how complex the source is (to be determined at a later date). A simple IDL implementation of equation 1 is provided with this report.

Next Steps

The members of POSSUM will continue to experiment with both model and real data in order to determine the value of ϵ_M and its dependence on instrumental noise and observational configuration.

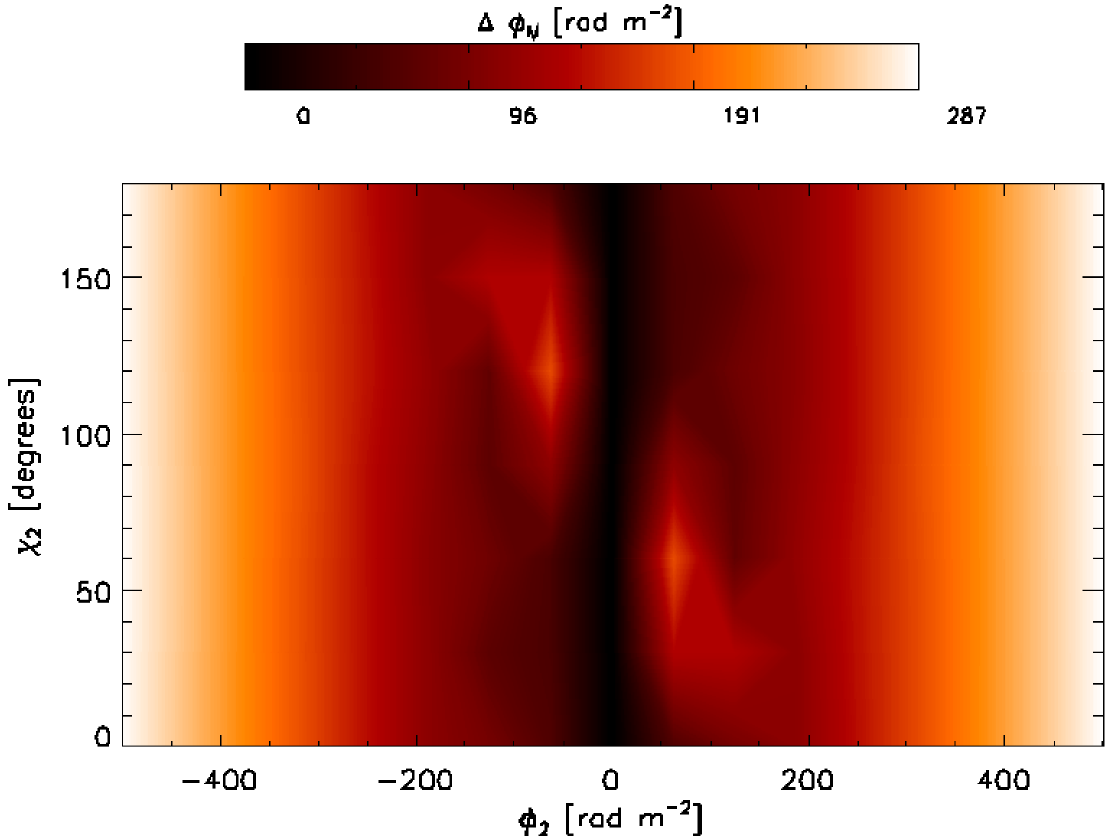

Figure 2: For each point on this plot, there is an RM spectrum with two components, one with amp=1, $\phi_1 = 0$, $\chi_1 = 0$, and the other with amp=0.5 and ϕ_2 and χ_2 given by the two axis. The colour is $\Delta\phi_M$ computed from the clean components of each spectrum, CLEANed with a gain of 0.1 to a threshold of 0.01 (no instrumental noise). The dark vertical line at $\phi_2 = 0$ is where the two components have combined into a single one, and $\Delta\phi_M = 0$. When the two components don't overlap, $\Delta\phi_M$ is non-zero, and the spectrum flagged as complex.