# Sparse Faraday Rotation Measure Synthesis

Andrecut et al. (2012)

The model of $F(\phi)$ is therefore characterized by a uniform grid, $\phi_m = -\phi_{win} + m\phi_R$, $m = 0, 1, ..., M-1$, and a vector $z = [z_0, z_1, ..., z_{M-1}] \in \mathbb{C}^M$, which is assumed sparse, i.e. it has a small number of non-zero components, corresponding to the complex amplitudes of the sources located on the $\phi_m$ grid. For example, a thin source with the amplitude $z_m$, located at $\phi_m$ , will be approximated by the product of $z_m$ with a Dirac function $\delta(\phi - \phi_m)$, while a thick source will be characterized by a contiguous set of non-zero amplitudes in the vector $z$, which requires a different set of adaptive functions, capable of capturing their position and extensive support in the $\phi$ space. The goal of the analysis is to find the vector $z$, which is a discrete approximation of the Faraday dispersion function $F(\phi)$, from the measurements $\tilde{Q}_n$ and $\tilde{U}_n$, $n = 0, 1, ..., N-1$.

$$\min_z \|z\|_1 \quad \textit{subject to} \quad \|W\Psi z - \tilde{p}\|_2^2 \leq (\beta\sigma)^2.$$

# Optimization

$$\min_{\xi}\left[\frac{1}{2}\left\|\Gamma\xi-\tilde{p}\right\|_2^2+\alpha\left\|\xi\right\|_1\right]$$

Goodness of fit Sparseness

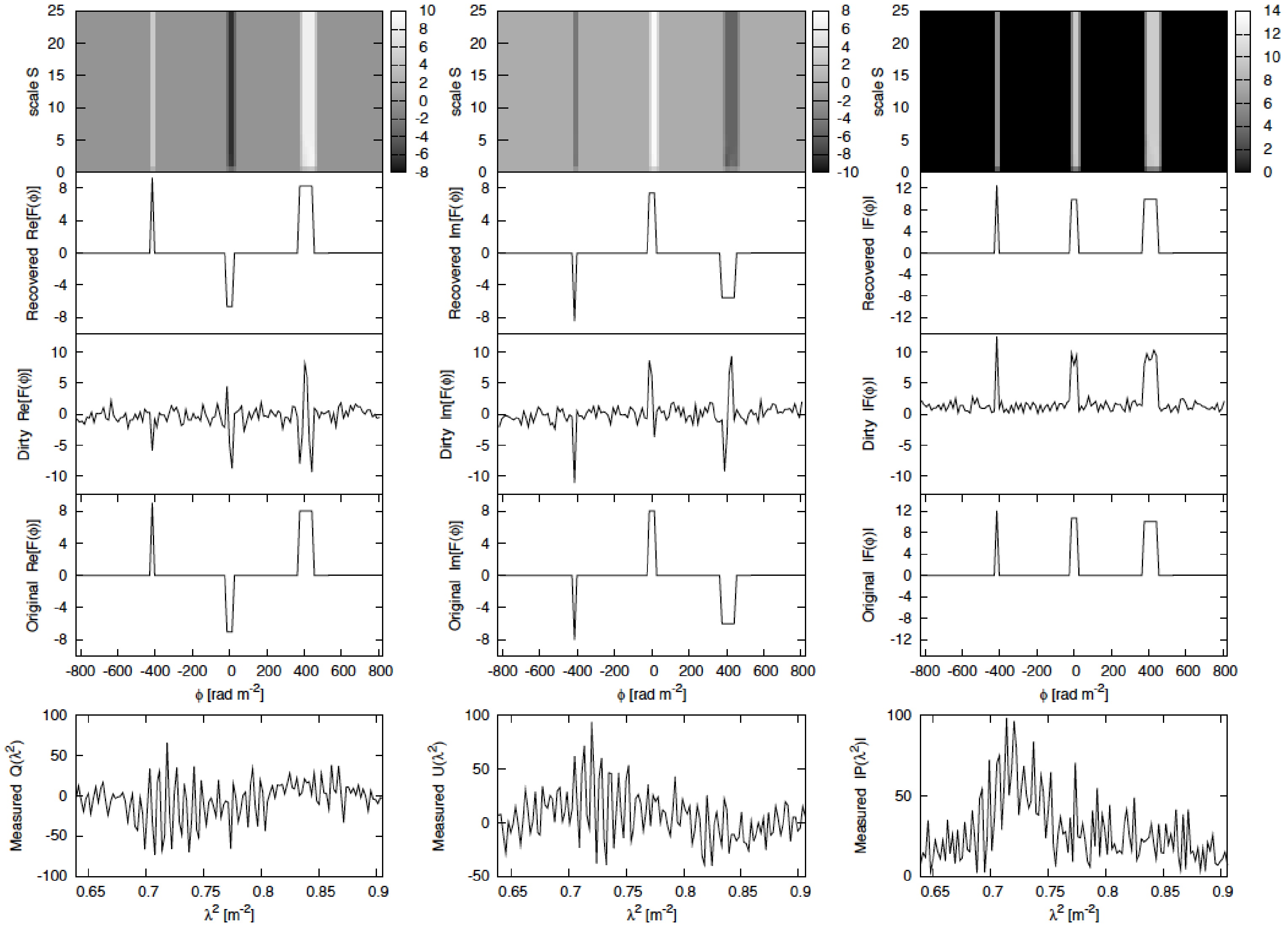


Fig. 1.— WSRT experiment layout, noiseless exact sampling case: $M = N = 126$ and $\phi_R = \delta\phi = 12.990\,\mathrm{rad\,m^{-2}}$. The figure is bottom-up organized: the bottom row is the

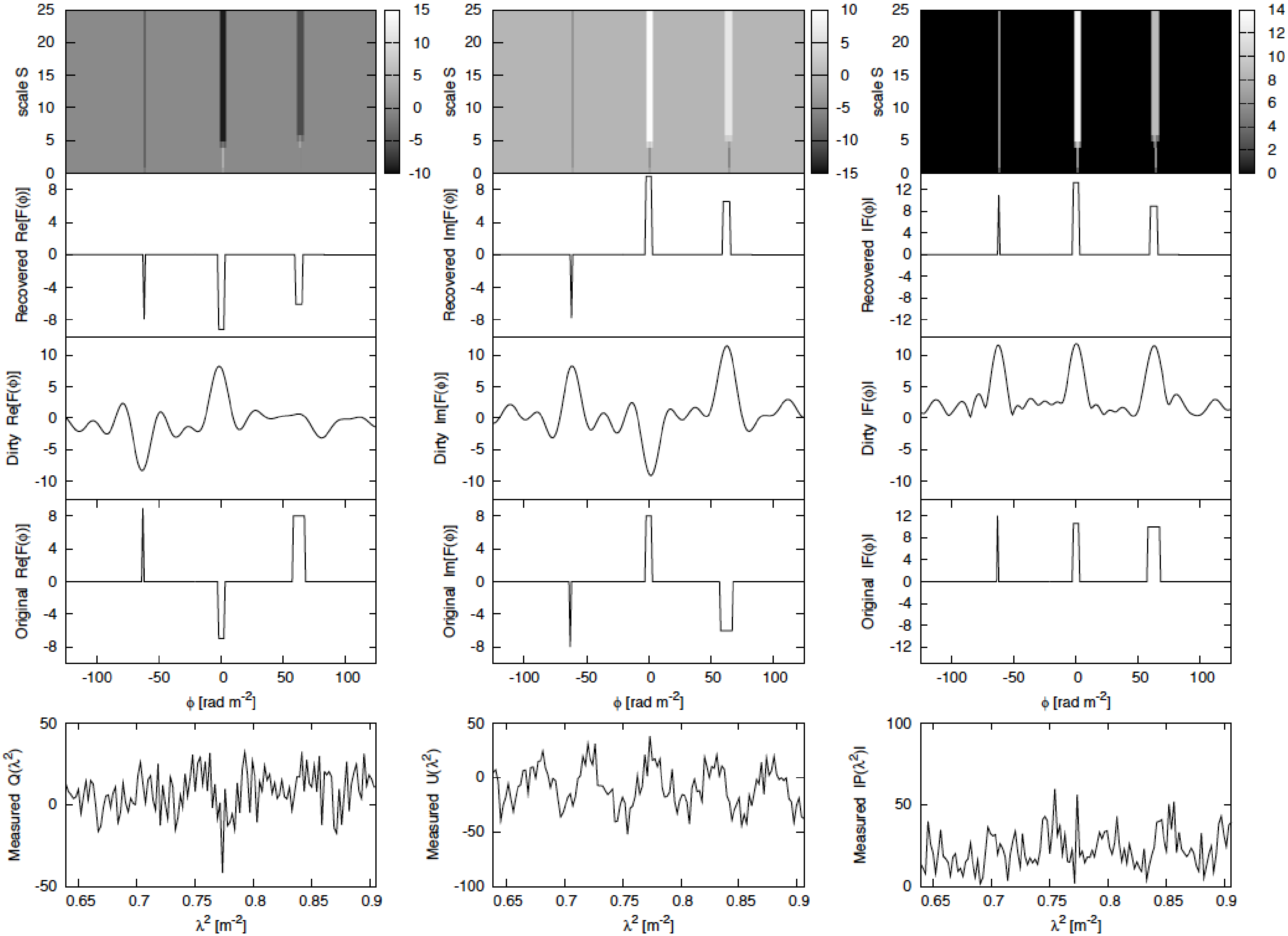


Fig. 2.— WSRT experiment layout, noisy sampling case ($\sigma = \sqrt{N}$): $N = 126$, $M = 252$ and $\phi_R = 1\,\mathrm{rad\,m^{-2}} \ll \delta\phi$.

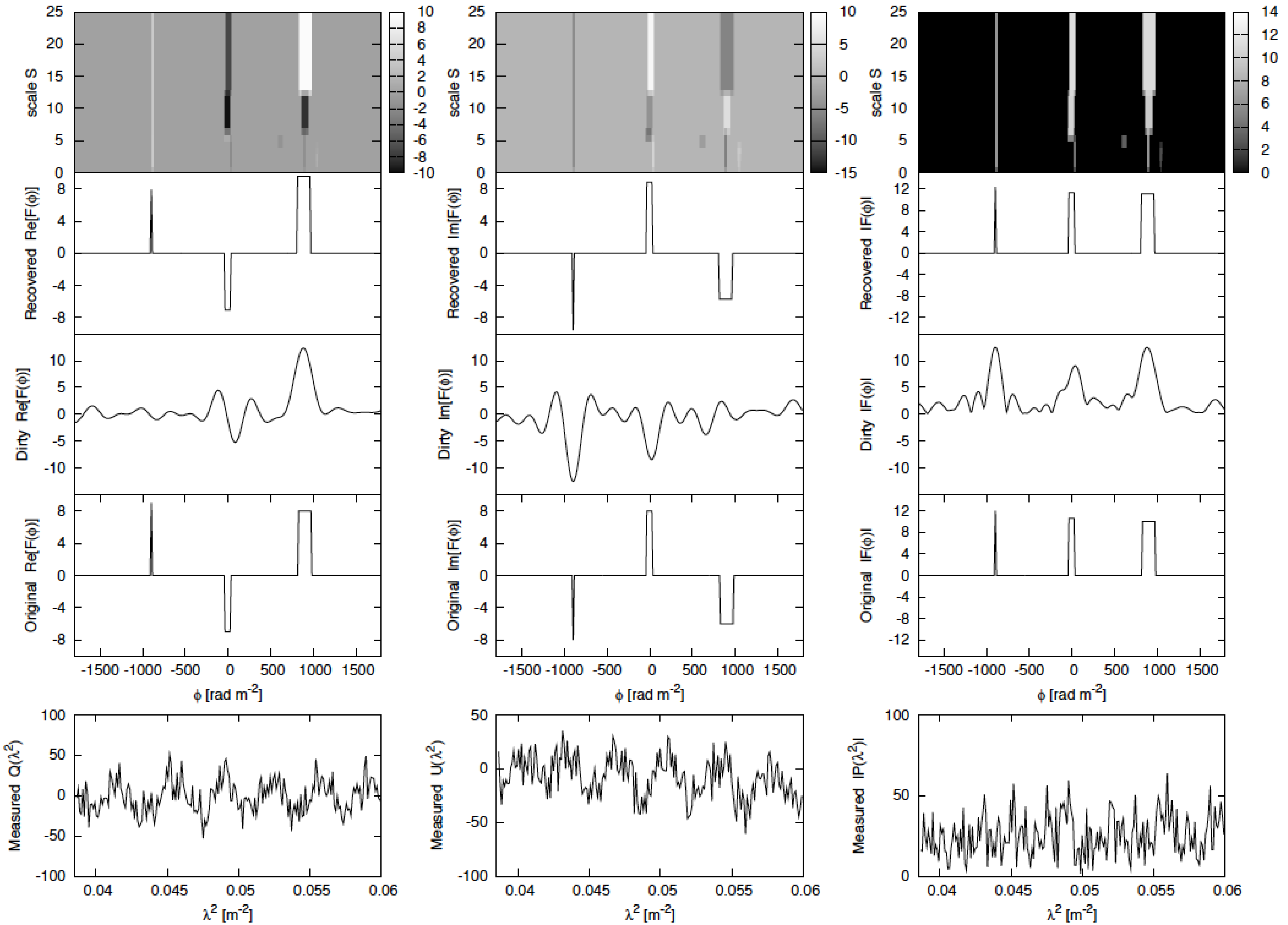


Fig. 4.— Arecibo experiment layout, noisy sampling case ($\sigma = \sqrt{N}$): $N = 200$, $M = 300$ and $\phi_R = 12\,\mathrm{rad\,m^{-2}} \ll \delta\phi$.

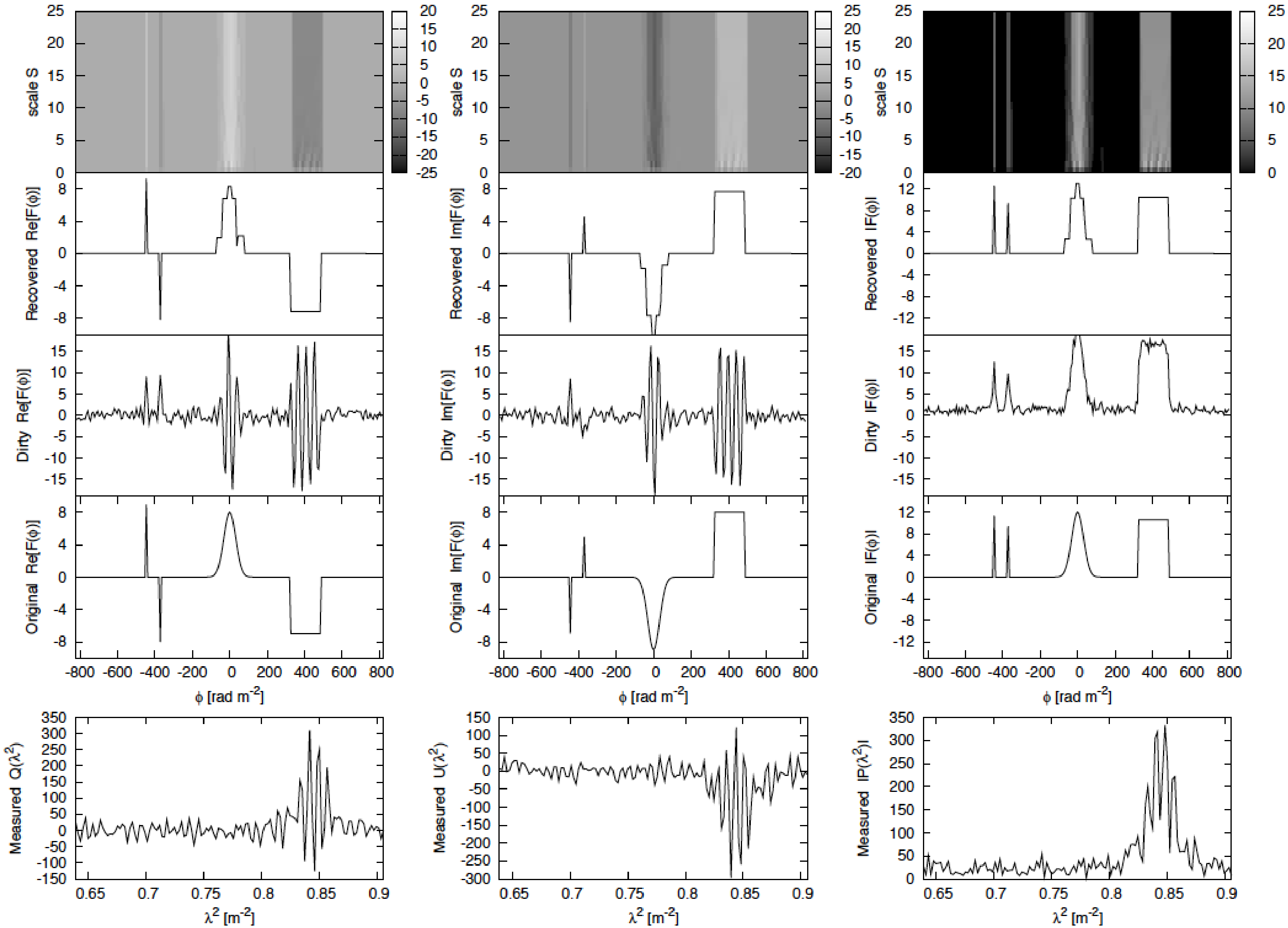


Fig. 7.— WSRT experiment layout, noisy sampling case ($\sigma = \sqrt{N}$): $N = 126$, $M = 220$ and $\phi_R = 7.440\,\mathrm{rad\,m^{-2}} = 0.57\delta\phi$.

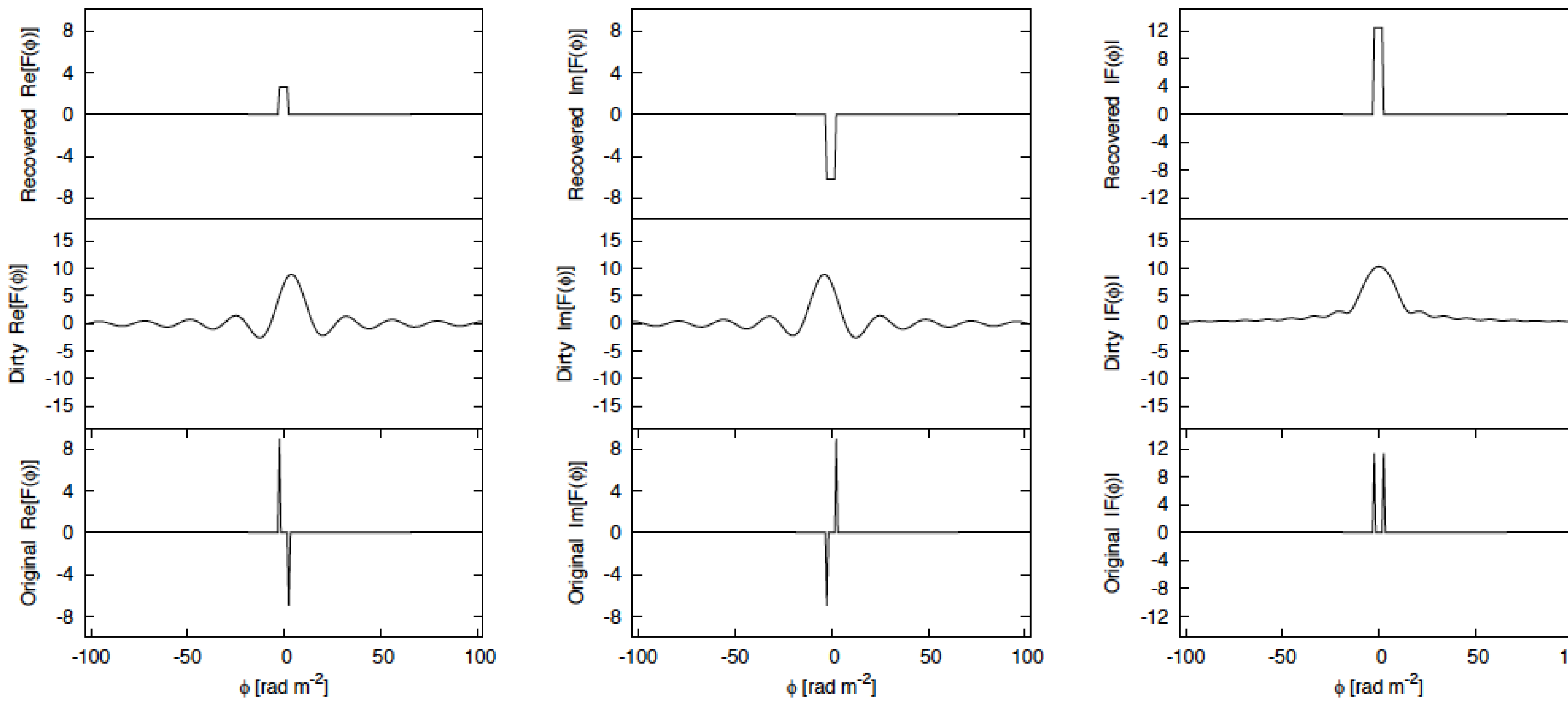


Fig. 9.— A typical response of the RM-MP algorithm for two Dirac components separated by $\Delta\phi_{in} = 5\phi_R = 4.06 \mathrm{rad\,m^{-2}} < \delta\phi = 12.99 \mathrm{rad\,m^{-2}}$. WSRT experiment layout, noiseless sampling case: $N = 126$, $M = 1008$ and $\phi_R = 0.812\,\mathrm{rad\,m^{-2}}$.